Swami Ramanand Teerth Marathwada University,

Nanded, Maharashtra, India

(UGC recognized, NAAC Re-accredited 'A' grade State University)

Biological Approach of Dynamical System

Research proposal submitted

In partial fulfilment of registration to

Ph.D. Degree

In the subject

Mathematics Faculty of Science

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2020

BIOLOGICAL APPROACH OF DYNAMICAL SYSTEM

➢ Abstract

We discuss the standard approach of dynamical system towards biomedical science. The purpose of this study is to meet the current and future needs for the interaction between various science and technology area on the one hand and dynamical system on the other hand. We discuss various models for interacting populations like Predator – Prey models, Lotka – Voltera system, Realistic Predator – Prey model, Continuous Population models for single species, Discrete Population models for single species etc. The aim of this study is to develop models which capture the essence of various interactions allowing the outcomes to be more fully understood. It has very broad perspective to analyse the biomedical issues using one of the most important branch of mathematics. The main objective of this study is to elaborate dynamical tools and this type of study will definitely act as bridge between biomedical sciences and mathematical sciences.

➢ Introduction

Dynamical system is the study of the longterm behaviour of evolving systems.The modern theory of dynamical systems originated at the end 19^{th} century with fundamental questions concerning the stability and evolution of the solar system .Attempts to answer those questions led to the development of rich and powerful field with applications to Physics,Biology, meteorology, astronomy,economics and other areas.

In recent years, dynamical system has had many applicationsto science and engineering. Some of which have gone under the related headings of chaos theory or nonlinear theory. The dynamical systems have two types continuous and discrete dynamical system.If the time in the equation is implicit then is called autonomous equation.Further if the time is explicit then it is called non autonomous.

Mathematics has always benefitted from its involvement with developing sciences.Each successive interaction revitalises and enhances the field.Biomedical sciences are clearly the premier science of the foreseeable futures.Mathematical biology is a fast growing well recognised and the most exciting modern application of mathematics.The increasing use of mathematics in biology is inevitable as biology becomes more quantitative.

The biological mathematical model becomes theme for the dynamical systems. So the biological model studied theoretically and numerically. The complexity of biological sciences makes interdisciplinary involvement essential. For the mathematicians biology opens up new and exciting brancheswhile for the biologists' mathematical modelling offers another research tools.The level of mathematics needed in collaborative biomedical research varies from the very simple to the sophisticated.

From a mathematical point of view, the art of good modelling relies on:

1. A sound understanding of biological problem.
2. A realistic mathematical representation of the important biological phenomena.
3. Finding useful solutions preferably quantitative.
4. A biological interpretation of mathematical results in terms of insights and predictions.

➢ History and Mathematical background of Dynamical System.

Many people regard French mathematician Henri Poincare as the founder of dynamical system. Poincare published two classical monographs, "New methods of Celestial Mechanics"(1892-1899) and "Lectures on Celestial Mechanics" (1905-1910). He successfully applied the results to the problem of the motion of three bodies and studied in detailed the behaviour of solutions (frequency, stability, asymptotic). These papers included the Poincare recurrence theorem, which states that certain systems will, after a sufficiently long but finite time , return to a state very close to the initial state.

Aleksandr Lyapunov developed many important approximation methods in 1899 which make it possible to define the stability of sets of ordinary differential equation and also contributed the modern theory of the stability of a dynamical system.

In 1913, George David Birkhoff proved Poincare's "Last Geometric Theorem", a special case of the "three body theorem". In 1927 Birkhoff published dynamical system. Birkhoff's most durable result has been his 1931 discovery of what is now called "Ergodic Theorem".

Stephen Smale made significant advances as well.His first contribution is the Smale horeshoe that jumpstarted significant research in dynamical system.

Oleksandr My kolaiovych Sharkovsky developed Sharkovsky's theorem on the periods of discrete dynamical system in 1964. One of the implications of the theorem is that if a discrete dynamical system on the real line has a periodic point of period 3, then it must have periodic points of every other period.

In the late 20th century, Palestinian mechanical engineer Ali H. Nayfeh applied non-linear dynamics in mechanical and engineering systems. His pioneering work in applied nonlinear dynamics has been influential in the construction and maintenance of machines and structures that are common in daily life such as ships, cranes, bridges, buildings, jet engines, rocket engines, aircraft and spacecraft.

➢ Recent works on the proposed topic

1. B.F. Finkenstadt , B.T. Grenfell(2002)

(Time Series modelling of childhood diseases: a dynamical system approach)

A key issue in the dynamical modelling of epidemics is the synthesis of complex mathematical models and data by means of time series analysis. An approach focussing On well documented case of measles was discussed.The authors proposed the use of discrete time epidemic model comprising the infected and susceptible class as state variables.The model used a discrete time version of the susceptible exposed infected recovered type epidemic model which can be fitted to be observed diseases incidence time series .Authors described a method for reconstructing the dynamics of the susceptible class,which is an unobserved state variable of the dynamical system. The model provides a remarkable fit to the data on case reports of measles in England and Wales from 1944 to 1964. Moreover its systematic part explains the well documented predominant biennial cyclic pattern .Authors studied the dynamic behaviour of the time series model and showed the episode of annual cyclicity, which have not previously been explained quantitatively arise as a response to a quicker replenishment of the susceptible class during the baby boom,around 1947.

2. Xinzhu Meng and Lansun Chen (2008)

(Global dynamical behaviours for an SIR epidemic model with time delay and pulse Vaccination)

In this paper authors proposed a delay SIR epidemic model with impulsive effect and analyse the global dynamics behaviour of the model. Using the discrete system determined by the stroboscopic map and showed there exists an infection free periodic solution in the model. The authors proved that the infection free periodic solution is global attractive when the period of impulsive effect is less than some critical value. The condition for the permanence of population for epidemic model with pulse vaccination is given which means the epidemic disease to spread around. Authors concluded that time delay and pulse bring great effects on the dynamic of the model. It illustrated theoretically that we should look carefully after the infectious patients to shorten the infections period. Lastly the biologic discussion and strategy for the elimination of infectious diseases are also given. In this paper the main feature is that authors introduced time delay together with pulse into epidemic model, and investigated their effects on the dynamics of model.

Since the investigations of impulsive delay differential equations are inchoate, the literature on delay epidemic models with pulse vaccination is not extensive. In this paper, we propose a delay SIR epidemic model with impulsive effect and analyse the global dynamics behaviour s of the model. Using the discrete dynamical system determined by the stroboscopic map, we show that there exists an ’infection-free’ periodic solution in the model. Further, we prove that the ’infection-free’ periodic solution is globally attractive when the period of impulsive effect is less than some critical value. The condition for the permanence of population for epidemic model with pulse vaccination is given, which means the epidemic disease to spread around. We conclude that time delay and pulse bring great effects on the dynamics of the model. It illustrated theoretically that we should look carefully after the infectious patients to shorten the convalescence period (or infections period). Lastly, the biologic discussion and strategy for the elimination of infectious diseases are also given. In this paper, the main feature is that we introduce time delay together with pulse into epidemic model, and we investigate their effects on the dynamics of model.

3. P. RajaSekharaRao , M. Naresh Kumar(2015)

(A dynamical model for Infectious diseases: The role of vaccination and treatment)

Understanding dynamics of an infectious diseases helps in designing appropriate strategies for containing its spread in a population. Recent mathematical models are aimed at studying dynamics some specific types of infectious diseases. In this paper author proposed a new model for infectious diseases spread having susceptible, infected and recovered populations and studied its dynamics in presence of incubation delay and relapse of the diseases.The influence of treatment and vaccination efforts on the spread of infection in presence of time delays are studied.Sufficient condition for local stability of the equilibria and change of stability are derived in various cases.The problem of global stability of the equilibria is studied for an important special case of the model.Simulations carried out in this study brought out the importance of treatment rate in controlling the diseases spread . It is observed that incubation delays have influence on the system even under enhanced vaccination .The present study has clearly brought out the fact treatment rate in presence of time delays would contain the diseases as compared to popular beliefs that eradication can only be done through vaccination.

4. Dian Grace Ludji, Paian Sianturi, Endar H. Nugrahani(2019)

(Dynamical system of the mathematical model for Tuberculosis with vaccination)

This research focused on the modification of deterministic mathematical models for tuberculosis with vaccination. The effect of giving vaccine is studied in the paper. It was done by adding vaccine compartments to people who were given the vaccine in the susceptible compartments. The population was divided into nine different groups. Those were susceptible (S),vaccine (V),new latently infected (E1),diagnosed latently infected (E2),undiagnosed latently infected (E3),undiagnosed actively infected(I),diagnosed actively infected with delay treatment(D_p),and treated (T).Basic reproduction number was constructed using next generation matrix.Sensitivity analysis was also conducted.The result showed the model comprises two equilibrium: disease free equilibrium (T_0) and endemic equilibrium(T*).It also showed that there is a relationship between R_0 and two equilibriums.

The diseases free equilibrium point is asymptotically stable local when $R_0 < 1$. The disease endemic equilibrium point is asymptotically stable local when $R_0 > 1$.

5. Sadiq Al-Nassir (May 27, 2020)

(The dynamics of biological models with optimal harvesting)

In this paper author discussed a concept of an equilibrium point of a dynamical system. A biological prey – predator model is analysed with a modification function growth in prey species. The conditions of the local stable and existence of all its equilibria are given. On extending the model to an optimal control problem, the discrete time version of Pontryagin's maximum principal is applied to solve the optimality problem.The characterization of the optimal harvesting variable and the adjoint variables are derived. Finally numerical simulations of various set of values of parameters are provided to confirm the theoretical findings.

6. Abul Kalam al Azad, Asif Mostafa Hussain (2020)

Modelling and analysis of the early growth dynamics of COVID-19 transmission)

In this exploratory research, the early growth dynamics of infection within a population is discussed by the authors based on real data. The study posits that the early growth in a homogenous population follows an exponential pattern motivating further rigorous quantitative treatment based on a number of analytical models such as logistic model, Richard's model, and Gompertz model– the acceleration pattern of the outbreak is ascertained from the daily inflection data, and regression analysis against population models yields dynamic growth indices which allow very accurate prediction of the successive outbreak size when calibrated continually with updated data.

The performance of the various models is evaluated with the real dataset. More, the basic reproduction number of the COVID-19 virus propagation in the community is estimated based on the on-set phase dataset using multi-compartmental epidemiological model. Also, the maximum infection size, infection doubling time and the scope of the herd immunity are also inferred for COVID-19 in a population.

➢ Scope of Study

The bridge between the mathematical sciences and biomedical sciences is heavily travelled. The correspondingly increased dialog between the disciplines has led to the establishment of the series**: Interdisciplinary Applied mathematics.** This will consist of monographs and high level texts from researchers working on the interplay between dynamical system and its biological applications.

➢ Objectives of proposed research work

1. To explore the world of dynamical systems.
2. To study the prerequisites: Calculus, Linear Algebra and Differential equations.
3. To study the concepts of state vectors and differentiate the dynamical world into the discrete and the continuous.
4. To explore many instances of dynamical system in the real world, specially the examples from biological background are to be discussed.
5. To discuss the biological problem.
6. To study the fixed points and stability of nonlinear systems .
7. To study the various population models and dynamics of infectious diseases.

➢ Research Methodology

The following steps will be including in the methodology.

1. To refer the relevant reference books on dynamical system.
2. To refer the various articles/papers which are based on the proposed topic.
3. To develop the biological model using various dynamical tools.
4. To learn the required mathematical software's like MATLAB and using the software's analyse the outcomes.
5. To find useful solution preferably quantitative.
6. To discuss biological interpretation of mathematical results in terms of insights and predictions.7.
7. To discuss a realistic dynamical representation of the important biological phenomena.

8. To discuss the global existence of the topic.
9. To publish research paper in reputed journals.

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